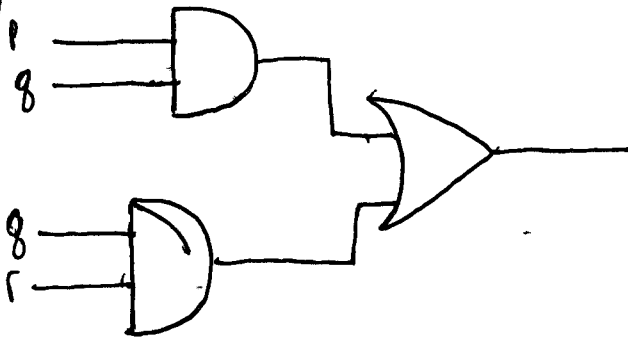


Problem One:

i) $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge (P \not\Rightarrow R)$

R	P	Q	$P \Rightarrow Q$	$Q \Rightarrow R$	$P \not\Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge (P \not\Rightarrow R)$
0	0	0	1	1	0	0
0	0	1	1	1	0	0
0	1	0	0	1	1	0
0	1	1	1	1	0	0
1	0	0	1	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	1	0
1	1	1	1	1	0	0



iii)

De Morgan's law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Therefore: $\neg(\neg p \wedge \neg q) \equiv p \vee q$

iv) ~~if~~ Combinations of a set = 2^k

Number of rows = 2^n .

Therefore, number distinct n -input truth tables = 2^{2^n}

v.) $(\neg P \wedge \neg R) \iff R$

Problem Two:

i.) All the time you can fool some of the people.

ii.) $\exists p \in P \forall t \in T : \text{CanFool}(u, p, t)$

iii.) S_1 implies that there is ~~some~~ at least one person at any given can be fooled. However, S_2 implies that there is a specific person who can be fooled at any given time. Therefore S_2 would be false if it wasn't the person, but S_1 ~~would~~ would still be true.

iv.) $\exists t \in T \forall p \in P : \text{CanFool}(u, p, t)$

v.) $\exists t \in T \exists p \in P : \text{CanFool}(u, p, t)$

vii) $\neg S_4 \Rightarrow (S_2 \wedge S_3)$ in english is:

You can fool all the people all the time implies that you can fool some people all the time and you can fool all the people some of the time.

This follows as the ability to fool all the people all the time would mean that you are both fooling some people all the time and fooling everybody sometimes. Because when both S_2 and S_3 are evaluated together like with $S_2 \wedge S_3$, ultimately you are fooling everyone all the time.

Problem Three:

i.) Proof: Contradiction

Assume: G isn't connected and ~~there are~~ at the degree of each vertex is at most $\frac{n}{2}$. This means in a graph with, for example, 4 vertices, but isn't connected, each vertex would have at most degree 2, however the graph ~~as simple~~ is simple, but this would mean vertices would ~~have multiple~~ potentially have multiple edges to the same vertex in its connected component. Contradiction!

~~ii.) Proof: Contrapositive~~

~~$\forall x \in \mathbb{Z}, x^7 - 2018x^3 + 2018x^2 \neq 1 \text{ then } x \neq 1$~~

~~Contrapositive: If $x = 1$~~

ii.) Proof: Contrapositive

$$\forall x \in \mathbb{Z}, x^7 - 2018x^3 + 2018x^2 \neq 1 \text{ then } x \neq 1$$

Contrapositive: If $x = 1$ then $x^7 - 2018x^3 + 2018x^2 = 1$

$$\Rightarrow (1)^7 - 2018(1)^3 + 2018(1)^2 = 1$$

$$\Rightarrow 1 - 2018 + 2018 = 1$$

$$\Rightarrow -2017 + 2018 = 1$$

$$\Rightarrow 1 = 1 \checkmark$$

iii.) Proof: Direct

$$\begin{aligned} n \text{ even} &\rightarrow n = 2k, k \in \mathbb{N} \rightarrow n(n-1) = 2k(2k-1) \\ &= 4k^2 - 2k = 2(2k^2 - k) \end{aligned}$$

$$\begin{aligned} n \text{ odd} &\rightarrow n = 2k+1, k \in \mathbb{N} \rightarrow n(n-1) = (2k+1)(2k+1-1) \\ &= 4k^2 + 2k = 2(2k^2 + k) \end{aligned}$$

Therefore: $n(n-1)$ is always even.